Math Diversion 980

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> Zathras is used to being beast of burden to other
> people's needs. Very sad life... Probably have
> very sad death. But, at least
> there is symmetry.
>
> —Zathras
>
> A character from *Babylon 5*

`Source: https://www.algebra.com/algebra`
`Title: Question 269702: A Mixed-Rate Problem`
`Presenter: Patrick`

1 Problem

The time it takes to do homework includes a fixed amount of time to prepare plus a constant amount of time per problem. If a student can do 5 homework problems in 40 minutes, and 10 problems in 70 minutes, how many minutes will 25 problems take?

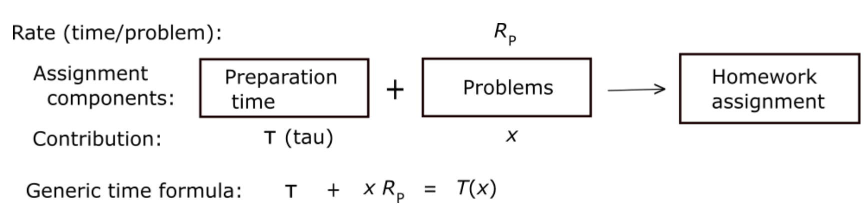

Figure 1. Preparation time τ and problem time enter on a different footing. Note: $R_F(x)$ means R_F times x, whereas $T(x)$ means T is a function of x.

2 Solution

What makes this problem different than most (maybe all) that we've encountered previously is that the parts of the total seem to enter into the problem as components on very different footings: The fixed time τ enters the figure as a time, not as a count, like its comrade x on the same line. Hence, we get the formula

$$\tau + R_P x = T(x)\,. \tag{1}$$

However, we could give 'preparation time' its own conversion factor $R_F = \tau$ minutes/assignment, where the F stands for 'fixed time'. Thus, $\tau = R_F \times 1$ [preparation]. Having done this, we could rewrite (1) as

$$R_F(1) + R_P x = T(x)\,. \tag{2}$$

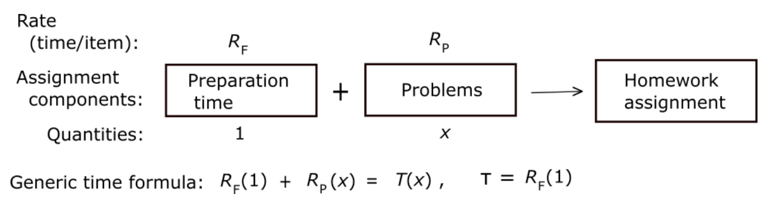

Figure 2. Preparation time τ and problem time enter on a different footing, but this time, however, the figure is now in standard form. Note: $R_F(x)$ means R_F times x, whereas $T(x)$ means that T is a function of x.

It's an interesting side point that if our generic 'student' messedup on his preparation and needed to do it over again, then the count for it would be 2 under the 'Preparation time' rectangle in Figure 2. If he messedup the prep time n times, we could account for this by altering Equation (2) to get

$$R_F \cdot n + R_P \cdot x = T(n, x)\,. \tag{3a}$$

This can be put in the standard form (where explicit referencing of the total's functional dependence is suppressed) as

$$R_F \cdot n + R_P \cdot x = T\,. \tag{3b}$$

We can visualize this generalization of the problem in the figure below.

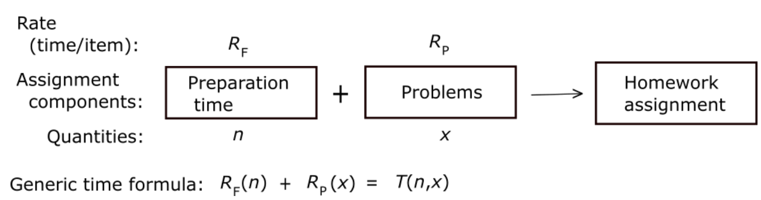

Figure 3. We have finally placed the figure in a truly general standard form by placing an unknown n beneath the 'Preparation time' rectangle.

By the way, we are told that this generic student is able to work each problem at a constant rate. This seems unlikely. So, even though the math doesn't need a re-interpretation of this time unit, commonsense does. Let's interpret this fixed time per problem as an average.

Anyway, from the information given, we can write the coupled equations:

$$\tau + R_P(5) = 40\,, \tag{4a}$$

$$\tau + R_P(10) = 70\,. \tag{4b}$$

The solution for R_P is 6 minutes/problem, and for τ is 10 minutes/assignment.