

# Math Diversion 981

P. Reany

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Source: <https://www.youtube.com/watch?v=WJfhnOMtsaY>  
Title: Solving a 'Harvard' University Entrance Exam Question  
Presenter: Math Explorer

## 1 Problem

Given the relation

$$\log_2 x + \log_3 x = 5, \tag{1}$$

solve for real values of  $x$ .

## 2 Solution

First, we'll perform an alpha transformation:

$$x = 2^\alpha, \tag{2}$$

so then (1) becomes

$$\log_2 2^\alpha + \log_3 2^\alpha = 5, \tag{3}$$

or

$$\alpha + \alpha \log_3 2 = 5, \tag{4}$$

or better yet,

$$\alpha(1 + \log_3 2) = 5. \tag{5}$$

But since  $1 = \log_3 3$ , the last equation becomes

$$\alpha(\log_3 3 + \log_3 2) = 5, \tag{6}$$

which becomes

$$\alpha(\log_3 6) = 5. \tag{7}$$

Hence,

$$\alpha = 5 \log_6 3. \tag{8}$$

Thus

$$x = 2^{5 \log_6 3}. \tag{9}$$