Math Diversion 983

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> Anything worth doing is
> worth doing well.
> — An Old Saying

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Source: https://www.youtube.com/watch?v=5C2VvcGEr4Y
Title: This High School Math Problem Will Test Your Skills
 Presenter: The Phantom of the Math
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1 Problem

This high-school trigonometry problem is presented in the figure below. Angle $\alpha = 60^\circ$.

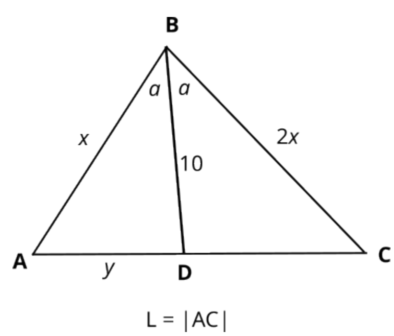

Figure 1. $\alpha = 60^\circ$. Solve for length L

2 Prerequisites

We will use the Law of Cosines and and the Angle Bisector Theorem, both of which can be looked up if needed.

$$\cos 60^\circ = 1/2, \quad \cos 120^\circ = -1/2. \tag{1}$$

3 Solution

Using the Law of Cosines for △ABC:

$$L^2 = x^2 + (2x)^2 - 2(x)(2x)(-1/2) = 7x^2\,, \tag{2}$$

or

$$L = \sqrt{7}x\,. \tag{3}$$

Then, using the Law of Cosines for △ABD:

$$y^2 = x^2 + (10)^2 - 2(x)(10)(1/2) = x^2 - 10x + 100\,. \tag{4}$$

From the Angle Bisector Theorem:

$$\frac{|BA|}{|AD|} = \frac{|BC|}{|CD|}\,. \tag{5}$$

Applied to this case, we have that

$$\frac{x}{y} = \frac{2x}{L-y}\,, \tag{6}$$

from which we conclude that

$$L = 3y\,. \tag{7}$$

On eliminating L between (2) and (7), we have that

$$9y^2 = 7x^2\,, \tag{8}$$

Multiplying (4) through by 9 and using (8), we get

$$9y^2 = 7x^2 = 9x^2 - 90x + 900\,, \tag{9}$$

which simplifies to the quadratic

$$x^2 - 45x + 450 = 0\,, \tag{10}$$

which has solutions

$$x = 15,\ 30\,. \tag{11}$$

And this presents two possible solutions for L:

$$L = 15\sqrt{7},\ 30\sqrt{7}\,. \tag{12}$$

The Presenter only got the lower value for L. I have tried to eliminate the larger value, but I failed and I'm out of time on it. I leave it to someone else to confirm or reject the larger value for L.