Math Diversion 984

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First things first...But not necessarily in that order.
— Doctor Who

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Source: https://www.youtube.com/watch?v=HVLVwJtMnF0
Title: Complex Trig Puzzle Explained! | P596
 Presenter: aplusbi
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1 Problem

Given the relation

$$\tan\theta - \sec\theta = i\,, \tag{1}$$

solve for real values of θ, and be general about it.

2 Prerequisites

When we derive one equation from another equation, when are they equivalent with respect to possible solutions? When both equations have the exact same solutions. So, when we clear of fractions, we may be changing the solution set. When we remove denominators, we remove the possibility of those denominators being zero.

3 Solution

First, we'll simplify the Given:

$$\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} = i\,. \tag{2}$$

Next, we multiply through by $\cos\theta$, to get

$$\sin\theta - 1 = i\cos\theta\,, \tag{3}$$

where we note now that (2) and (3) are not equivalent. What does really mean? It means that we may obtain solutions to the latter that may not apply to the former. Next, multiply through by i:

$$i \sin\theta - i = -\cos\theta \,, \tag{4}$$

which can be algebraically rewritten as

$$\cos\theta + i \sin\theta = i \,. \tag{5}$$

Using Euler's Identity, we can rewrite both sides, to get

$$e^{i\theta} = e^{i\pi/2} e^{2\pi i k} \,, \tag{6}$$

where $k \in \mathbb{Z}$. Furthermore, this last equation can be expressed as

$$e^{i\theta} = e^{i(\pi/2 + 2\pi k)} \,. \tag{7}$$

Hence, the 'solution' for θ is

$$\theta = \pi/2 + 2\pi k \,. \tag{8}$$

These solutions for θ are perfectly good for (3), but not for (1). But why? Because these solutions we got for θ will set the $\cos\theta = 0$. Hence, the Given equation has no solutions.