

# Math Diversion 985

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For unto you is born this day in the city of David  
a Saviour, which is Christ the Lord.  
— Luke 2:11  
(I don't know Christ's actual BD.)

Source: <https://www.youtube.com/watch?v=zM0fo9bV2II>  
Title: A Deceptively Hard Integral  
Presenter: Mental Math

## 1 Problem

Find the integral  $I$ :

$$I = \int_0^1 \frac{x^5}{1+x^2} dx. \quad (1)$$

## 2 Solution

My first impression as I gaze at an integrand like this is that I will likely need at least one variable substitution along the way.

Let's first try an obvious attempt at simplification:

$$I = \frac{1}{2} \int_0^1 \frac{x^4}{1+x^2} d(1+x^2). \quad (2)$$

Now, if we make the variable substitution:

$$u = 1 + x^2, \quad (3)$$

along with the appropriate change in limits, (2) becomes

$$I = \frac{1}{2} \int_1^2 \frac{u^2 - 2u + 1}{u} du. \quad (4)$$

This simplifies to

$$I = \frac{1}{2} \int_1^2 (u - 2 + u^{-1}) du. \quad (5)$$

So now we're ready to integrate:

$$I = \frac{1}{2} \left( \frac{1}{2} u^2 - 2u + \ln u \right) \Big|_1^2 = \frac{1}{2} \left( \ln 2 - \frac{1}{2} \right). \quad (6)$$