Math Diversion 986

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Truth, like oil, will in time rise to surface.
— Charlie Chan

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Source: https://www.youtube.com/watch?v=0aMU0q3w-mo
Title: A Very Nice Exponential Equation Can You Solve It?
 Presenter: Mental Math
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1 Problem

Given the relation

$$\left(\frac{x}{x+11}\right)^{x+11} = \frac{1}{2048}\,, \tag{1}$$

solve for real values of x.

2 Prerequisite

It helps to know that 2048 is a power of 2, namely

$$2048 = 2^{11}\,. \tag{2}$$

3 Solution

First, we'll perform a variable substitution:

$$u = x + 11\,, \tag{3}$$

so then (1) becomes

$$\left(\frac{u-11}{u}\right)^{u} = 2^{-11}\,, \tag{4}$$

or

$$\left(1 - \frac{11}{u}\right)^{u} = 2^{-11}\,. \tag{5}$$

Now we take the eleventh root of both sides:

$$\left(1 - \frac{11}{u}\right)^{u/11} = 2^{-1}\,. \tag{6}$$

What if we simplified a little more with $z = 11/u$, then we get

$$(1 - z)^{1/z} = \frac{1}{2}\,. \tag{7}$$

At this point, perhaps we should throttle down, take a deep breath, and hope that we can finish off this problem by inspection.[1] And I think we can! Let's try

$$z = -1\,. \tag{8}$$

Then (7) becomes

$$\big(1 - (-1)\big)^{1/(-1)} = \frac{1}{2}\,, \tag{9}$$

which works. Yay! Hence, $u = -11$, and therefore

$$x = u - 11 = -22\,. \tag{10}$$

[1] To 'solve a problem by inspection' just means to perform the calculations in your head.