Math Diversion 987

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Don't ever take a fence down until you
know the reason it was put up.
— Chesterton

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Source: https://www.youtube.com/watch?v=cgUtbvIQC3Q
Title: Can You Solve This Exponential Equation in 30 Seconds
 Presenter: Mental Math
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1 Problem

Given the relation

$$2^x + 2^{1-x} = 3\,, \tag{1}$$

solve for real values of x.

Note: We are teased to solve the problem by inspection, but if you can't, I'll demonstrate a more formal approach.

2 Solution

First, we'll perform a variable substitution:

$$y = 2^x\,, \tag{2}$$

so that (1) becomes

$$y + 2y^{-1} = 3\,. \tag{3}$$

From this we get the quadratic in y:

$$y^2 - 3y + 2 = 0\,, \tag{4}$$

which has the two roots

$$y = 1,\ 2\,. \tag{5}$$

Thus, the solutions for x are

$$x = \log_2 1 = 0,\ x = \log_2 2 = 1\,. \tag{6}$$