Math Diversion 988

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> Talent is cheaper than table salt. What separates
> the talented individual from the successful
> one is a lot of hard work.
> — Stephen King

`Source: https://www.youtube.com/watch?v=39eNat-v4og`
`Title: Only 1% of Students Solve This Logarithm Problem Correctly!`
`Presenter: NonsoMaths`

1 Problem

Given the relation

$$\log_x 10 + 2\log_{10x} 10 = 3\log_{100x} 10\,, \tag{1}$$

solve for the values of x.

2 Prerequisites

Hint: This problem is not difficult to those who are very familiar with the most common rules of using logarithms. So, it behooves the problem solver to get familiar with them.

3 Solution

We'll begin by converting the Given with a common logarithm rule:

$$\frac{\log 10}{\log x} + 2\frac{\log 10}{\log 10x} = 3\frac{\log 10}{\log 100x}\,, \tag{2}$$

where the logarithms are being taken base 10. And since $\log 10 = 1$, (2) becomes

$$\frac{1}{\log x} + 2\frac{1}{\log 10x} = 3\frac{1}{\log 100x}\,. \tag{3}$$

Using a couple more rules for logarithms, this last equation expands to

$$\frac{1}{\log x} + 2\frac{1}{1+\log x} = 3\frac{1}{2+\log x}\,. \tag{4}$$

And as is my custom, I will make a variable substitution at this point. Let

$$y = \log x\,, \tag{5}$$

then (4) becomes

$$\frac{1}{y} + 2\frac{1}{1+y} = 3\frac{1}{2+y}\,. \tag{6}$$

As is usual, we remove denominators, to get

$$(1+y)(2+y) + 2(y)(2+y) = 3(y)(1+y)\,. \tag{7}$$

Fortunately, there is a lot of cancellation we'll benefit from, and the solution for y is:

$$y = -1/2\,. \tag{8}$$

From (5) we have that $x = 10^y$, so that

$$x = 10^{-1/2} = \frac{1}{\sqrt{10}}\,. \tag{9}$$