Math Diversion 990

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People often overlook the obvious.
— Doctor Who

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Source: https://www.youtube.com/watch?v=Y8Xp1nPEycE
Title: The Real Deal | P589
 Presenter: aplusbi
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1 Problem

Given the relation

$$|z|^2 - 2\mathrm{Re}(iz) = 3\,, \tag{1}$$

solve for the locus of points $\{(a,b)\}$ satisfying (1), where

$$z = a + bi \quad \text{and} \quad |z| = \sqrt{a^2+b^2}\,. \tag{2}$$

Think of the variables a and b as taking on the roles of x and y, respectively, in the complex plane.

2 Solution

So, $\mathrm{Re}(iz) = \mathrm{Re}(i(a+bi)) = -b$. Then, using this result and substituting values from (2) into (1), we have that

$$(a^2+b^2) + 2b = 3\,. \tag{3}$$

Let's try to maneuver this into a standard form.

$$a^2 + b^2 + 2b + 1 - 1 = 3\,, \tag{4}$$

which becomes

$$a^2 + (b+1)^2 = 4 = 2^2\,. \tag{5}$$

So, this describes a circle of radius 2 centered at the point $(0,-1)$.