Math Diversion 991

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December 28, 2025

No mystery is closed to an open mind.
— Tim White
Sightings TV show

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Source: https://www.youtube.com/watch?v=HQxPzY3C6dg
Title: Two Equations, Infinite Curiosity
 Presenter: SyberMath
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1 Problem

Given the relations

$$3\sin A + 4\cos B = 6\,, \tag{1a}$$
$$4\sin B + 3\cos A = 1\,, \tag{1b}$$

where A and B are interior angles of triangle ABC, solve for the values of A, B.

2 Prerequisites

$$\sin^2\theta + \cos^2\theta = 1\,. \tag{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta\,. \tag{3}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\,. \tag{4}$$

So why am I including an identity for tangents? Because WolframAlpha gave its solutions in terms of inverse tangents, so I can compare.

3 Solution

I don't have a full solution, and I've run out of time, but perhaps what I do have will be useful to the reader.

My first attempt at a solution I tried to eliminate one of the variables, but that didn't go well. My second attempt was as follows:[1]

Add (1a) and (1b) together, to get

$$3(\sin A + \cos A) + 4(\sin B + \cos B) = 7\,. \tag{5}$$

So, this equation has a possible solutions for A and B, if we can find solutions to the coupled equations[2]

$$\sin A + \cos A = 4/3\,, \tag{6a}$$
$$\sin B + \cos B = 3/4\,, \tag{6b}$$

subject to the constraint that $A + B < 180°$.

So, we proceed to deal with a lot of messy trig stuff, begi9nning by squaring (6a):

$$(\sin A + \cos A)^2 = \frac{16}{9}\,, \tag{7}$$

which simplifies down to

$$2 \sin A \cos A = \frac{7}{9}\,. \tag{8}$$

On using (3), we have that

$$2 \sin A \cos A = \sin 2A\,. \tag{9}$$

And now we're really getting somewhere.

$$\sin 2A = \frac{7}{9}\,. \tag{10}$$

So, even if calculating A from an inverse sine makes sense, I went on to get the inverse tangent: Hence,

$$\cos 2A = \pm\frac{4\sqrt{2}}{9}\,, \tag{11}$$

and so,

$$\tan 2A = \pm\frac{7}{4\sqrt{2}}\,. \tag{12}$$

To find the tangent of the angle, we go back to (4)

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}\,. \tag{13}$$

[1] I also tried the naive $\sin A + \cos A = \sin B + \cos B = 1$, but that didn't work.

[2] You can try these in WolframAlpha: $\sin A + \cos A = 4/3,\ \sin B + \cos B = 3/4$, solve for A,B

On combining these last two equations and solving for the tangent, we get[3]

$$\tan A = \pm\frac{\sqrt{2}}{7} \pm \frac{9}{7}\,, \tag{14}$$

where the $\pm$ signs are uncoupled, and thus give four possible solutions. We note that $\tan A > 0$ if $0 < A < \pi/2$, and that $\tan A < 0$ if $\pi/2 < A < \pi$.

So what about angle B? We have to perform a similar calculation as we did for angle A, using (6b), and then find all combinations of the pair of angles that satisfy the constraint that $A + B < 180°$. And I leave that to the interested reader to perform. (Or, maybe I'll come back to it someday.)

[3]This gives an expression for tangent A that is at least of similar form to the WolframAlpha solutions.