

Math Diversion 993

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The ignorant always loud in argument.

— Charlie Chan

Source: <https://www.youtube.com/watch?v=j1zdTAsfbbI>

Title: This Radical Equation is EASIER Than it Looks

Presenter: NonsoMaths

1 Problem

Given the relation

$$\sqrt{2x^2 - 7x + 1} - \sqrt{2x^2 - 9x + 4} = 1, \quad (1)$$

solve for real values of x .

2 Prerequisites

I'll be using the Unipodal algebra to solve this. If you need some background on it, I have a writeup on it. One particular is that $u^2 = 1$. Also,

$$u_+^2 = u_+ \quad \text{and} \quad u_-^2 = u_-. \quad (2)$$

3 Solution

I'll take as my first unipode,

$$a = \sqrt{2x^2 - 7x + 1}u_+ + \sqrt{2x^2 - 9x + 4}u_- \quad (3a)$$

$$= \frac{1}{2}[\sqrt{2x^2 - 7x + 1} + \sqrt{2x^2 - 9x + 4}] + \frac{1}{2}[\sqrt{2x^2 - 7x + 1} - \sqrt{2x^2 - 9x + 4}]u \quad (3b)$$

$$= \frac{1}{2}\beta + \frac{1}{2}u, \quad (3c)$$

where we used (1) and where

$$\beta \equiv \sqrt{2x^2 - 7x + 1} + \sqrt{2x^2 - 9x + 4}. \quad (4)$$

Next, we square a :

$$a^2 = \left(\frac{1}{4}\beta^2 + \frac{1}{4}\right) + \frac{1}{2}\beta u. \quad (5)$$

So, let's recap. We got this last form for a^2 by first converting a to standard form and then squaring. Next, we first square a and then convert that to standard form. After that, we'll equate, respectively, the complex and uniplex parts of both forms.

$$a^2 = (2x^2 - 7x + 1)u_+ + (2x^2 - 9x + 4)u_- \quad (6)$$

$$= \frac{1}{2}(4x^2 - 16x + 5) + \frac{1}{2}(2x - 3)u. \quad (7)$$

So, we begin by equating the scalar parts, to get

$$\left(\frac{1}{4}\beta^2 + \frac{1}{4}\right) = \frac{1}{2}(4x^2 - 16x + 5), \quad (8)$$

or simplifying a bit

$$\beta^2 + 1 = 2(4x^2 - 16x + 5). \quad (9)$$

And, on equating the unipotent parts, we have that

$$\frac{1}{2}\beta = \frac{1}{2}(2x - 3). \quad (10)$$

On eliminating β between Equation (9) and (10), we get (eventually)

$$x(x - 5) = 0, \quad (11)$$

which has solutions

$$x = 0 \quad \text{and} \quad x = 5. \quad (12)$$

Checking both of these possible solutions to (1), we get only that $x = 5$ works.