Math Diversion 996

P. Reany

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He that answereth a matter before he heareth it,
it is folly and shame unto him.
— Proverbs 18:13

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Source: https://www.youtube.com/watch?v=kl8uoJt2iDw
Title: This Log Equation in Different Bases Stumps Students
 Presenter: NonsoMaths
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1 Problem

Given the relation

$$\log_8(9x) = \log_9(8x)\,, \tag{1}$$

solve for real values of x.

2 Solution

By a familiar logarithm lemma, we can rewrite (1) as

$$\frac{\log(9x)}{\log 8)} = \frac{\log(8x)}{\log 9}\,. \tag{2}$$

Now, if we let

$$\gamma = \frac{\log 9)}{\log 8}\,, \tag{3}$$

then (2) becomes

$$\gamma(\log 9 + \log x) = \log 9 + \log x\,. \tag{4}$$

On solving for $\log x$ with only algebra, we have that

$$\log x = \frac{\log 8 - \gamma \log 9}{\gamma - 1} = -\frac{(\log 9)^2 - (\log 8)^2}{\log 9 - \log 8} = -(\log 9 + \log 8) = -\log 72\,.$$

Therefore

$$x = \frac{1}{72}\,. \tag{5}$$