

# Math Diversion 997

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After a time, you may find that having is not  
so pleasing a thing after all as wanting. It is  
not logical, but is often true.

— Spock

Source: <https://www.youtube.com/watch?v=8rexi2JUk6M>

Title: A Nice Algebra Problem | Math Olympiad

Presenter: SALogic

## 1 Problem

Given the relation

$$2^{x^2} = \frac{10}{5^x}, \quad (1)$$

solve for real values of  $x$ .

## 2 Solution

By taking the logarithm base 2 across this equation, we get

$$\begin{aligned} x^2 &= \log_2 10 + \log_2(5^{-x}) \\ &= \log_2(2 \cdot 5) - x \log_2 5 \\ &= \log_2(2 \cdot 5) - x \log_2 5 \\ &= \log_2 2 + \log_2 5 - x \log_2 5. \end{aligned} \quad (2)$$

On remembering that  $\log_2 2 = 1$  and setting  $\alpha = \log_2 5$ , then (2) becomes

$$x^2 + \alpha x - (1 + \alpha) = 0, \quad (3)$$

which has roots

$$x = 1, -\alpha - 1, \quad (4)$$

or

$$x = 1, -\log_2 5 - 1. \quad (5)$$