

Math Diversion 1000

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There is much you have to learn. You must explore; you
must reach out. Go...and give thought to the
the mysteries of the universe.
— The Galaxy Being
(An early proponent of
Life-Long Learning)

Source: <https://www.youtube.com/watch?v=QvW1zHap2D4>
Title: A nice radical maths olympiad question
Presenter: Rashel's Classroom

1 Problem

Given the relation

$$\sqrt{39-x} + \sqrt{7-x} = 8, \quad (1)$$

solve for real values of x .

2 Solution

I'll take as my first unipode,

$$a = \sqrt{39-x}u_+ + \sqrt{7-x}u_- \quad (2a)$$

$$a^2 = (39-x)u_+ + (7-x)u_- \quad (2b)$$

$$= \frac{1}{2}[(39-x) + (7-x)] + \frac{1}{2}[(39-x) - (7-x)]u \quad (2c)$$

$$= (23-x) + 16u. \quad (2d)$$

So, let's recap. We got this last form for a^2 by first squaring a to get a^2 , and then converting that to standard form. Next, we'll convert a to standard form and then square that. After that, we'll equate, respectively, the complex and uniplex parts of both forms.

$$a = \sqrt{39-x}u_+ + \sqrt{7-x}u_- \quad (3a)$$

$$= \frac{1}{2}[\sqrt{39-x} + \sqrt{7-x}] \quad (3b)$$

$$+ \frac{1}{2}[\sqrt{39-x} - \sqrt{7-x}]u \quad (3c)$$

$$a = 4 + \frac{1}{2}Bu. \quad (3d)$$

where we used (1) and where

$$B \equiv \sqrt{39-x} - \sqrt{7-x}. \quad (4)$$

Next, we square a :

$$a^2 = (16 + \frac{1}{4}B^2) + 4Bu. \quad (5)$$

On equating the uniplex part of (2d) with the uniplex part of (6), we have that

$$16 = 4B, \quad (6)$$

hence,

$$B = 4. \quad (7)$$

Now, if we equate the complex part of (2d) with the complex part of (6), we have that

$$23 - x = 16 + \frac{1}{4}B^2. \quad (8)$$

On solving these last two equation together for x , we get

$$x = 3. \quad (9)$$

And this value of x does satisfy Eq. (1).