Math Diversion 1001

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> It is not enough to be in the right place at the right time. You should also have an open mind at the right time.
> — Paul Erdós

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Source: https://www.youtube.com/watch?v=gJeDWfsgS9Y
Title: This Quadratic Leads to a Wild Power! l P590
 Presenter: aplusbi
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1 Problem

Given the relation

$$z^2 + 2z + 2 = 0\,, \tag{1}$$

find the values of

$$\phi = z^{11} + z^{-11}\,. \tag{2}$$

Look for solutions whose argument lies between 0 and 2π.

2 Solution

We can use the quadratic formula to solve for the roots to (1)

$$z_\pm = -1 \pm i = -\sqrt{2}\frac{1 \pm i}{\sqrt{2}}\,. \tag{3}$$

But we can rewrite z into polar form:

$$z_\pm = -\sqrt{2}e^{\pm\pi i/4}\,. \tag{4}$$

So then

$$\begin{aligned}
\phi &= z_{\pm}^{11} + z_{\pm}^{-11} \\
&= (-\sqrt{2}e^{\pm\pi i/4})^{11} + (-\sqrt{2}e^{\pm\pi i/4})^{-11} \\
&= -2^{11/2}[e^{\pm 11\pi i/4}] + -2^{11/2}[e^{\mp 11\pi i/4}] \\
&= -2^{11/2}[e^{11\pi i/4} + e^{-11\pi i/4}] \\
&= -2^{11/2}[e^{-\pi i/4} + e^{\pi i/4}] \\
&= -2^{11/2}2\frac{e^{-\pi i/4} + e^{\pi i/4}}{2} \\
&= -2^{13/2}\cos\pi/4 \\
&= -2^{13/2}\frac{1}{\sqrt{2}} \\
&= -2^{12/2} = -2^6 = -64\,. \qquad (5)
\end{aligned}$$

However, this is not the answer that either the Presenter or WolframAlpha got, so it seems that my answer is wrong. Perhaps you can find the error.