Math Diversion 1003

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January 5, 2026

A mathematician is a machine for turning coffee into theorems.
— Alfréd Rényl

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Source: https://www.youtube.com/watch?v=DLmuvWL_RCg
Title: Can You Solve This Exponential Equation Without Guessing?
 Presenter: Math Mastery with Amitesh
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1 Problem

Given the relation

$$16^{x^2+y} + 16^{x^2+y} = 1\,, \tag{1}$$

find real solutions for (x, y).

So, can I solve this exponential equation without guessing?

Unfortunately, I don't have the time right now to find out, so I'm going to guess.

2 Solution

Now, I know that

$$\tfrac{1}{2} + \tfrac{1}{2} = 1\,, \tag{2}$$

and since $16^{-1/4} = 1/2$, then

$$16^{-1/4} + 16^{-1/4} = 1\,. \tag{3}$$

So, possible solutions come from the coupled equations

$$x^2 + y = -1/4\,, \tag{4}$$

$$x + y^2 = -1/4\,. \tag{5}$$

On setting these equal, we get

$$x^2 + y = x + y^2\,, \tag{6}$$

which leads us to conclude that $x = y$. Using this in (4) and solving for x, we get $x = -1/2$. Hence, our solution set is

$$\{(-1/2, -1/2)\}\,. \tag{7}$$