Math Diversion 1004

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A mathematician is a machine for turning coffee into theorems.
— Alfréd Rényl

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Source: https://www.youtube.com/shorts/xT2QV8LV2Z4
Title:Interesting Exponential Equation
 Presenter: drpkmath
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1 Problem

Given the relation

$$t^t = 49\,, \tag{1}$$

find real solutions for t.

2 Solution

First, we take the natural logarithm across the Given equation.

$$t \ln t = \ln 49 = \ln 7^2 = 2 \ln 7\,. \tag{2}$$

Next, we take the Lambert W function across this equation[1]

$$\ln t = W_n(2 \ln 7)\,. \tag{3}$$

Finally, we take the inverse logarithm, to get

$$t = e^{W_n(2 \ln 7)}\,. \tag{4}$$

Since the graph of the two functions $y = t^t$ and $y = 49$ shows their intersection as single point, we conclude that we must take $n = 0$, thus

$$t = e^{W(2 \ln 7)}\,. \tag{5}$$

[1] If you're having trouble following the rest of the proof, see my writeup on the Lambert W function.