

# Math Diversion 1005

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Mathematics is the art of reducing any  
problem to linear algebra.  
— William Stein

Source: [https://www.youtube.com/watch?v=k0B0Cf7CO\\_g](https://www.youtube.com/watch?v=k0B0Cf7CO_g)  
Title: The Square Root Trap!  
Presenter: SyberMath

## 1 Problem

Given the relation

$$x - \sqrt{a - x^2} = 1, \quad (1)$$

solve for real values of  $x$ .

## 2 Solution

I'll take as my first unipode,

$$b = xu_+ + \sqrt{a - x^2}u_- \quad (2a)$$

$$= \frac{1}{2}[x + \sqrt{a - x^2}] + \frac{1}{2}[x - \sqrt{a - x^2}]u \quad (2b)$$

$$= \frac{1}{2}B + \frac{1}{2}u. \quad (2c)$$

where we used (1) and where

$$B \equiv x + \sqrt{a - x^2}. \quad (3)$$

On squaring  $b$  from line (2c), we have that

$$b^2 \equiv \frac{B^2}{4} + \frac{1}{4} + \frac{B}{2}u. \quad (4)$$

So, let's recap. We got this last form for  $b^2$  by first converting to standard form and then squaring  $b$ . Next, we'll convert  $b$  from line (2a) to standard form

and then square that. After that, we'll equate, respectively, the complex and uniplex parts of both forms of  $b^2$ .

$$b^2 = x^2 u_+ + (a - x^2) u_- \quad (5a)$$

$$= \frac{a}{2} + [x^2 - \frac{a}{2}] u \quad (5b)$$

On equating the uniplex part of (5b) with the uniplex part of (4), we have that

$$x^2 - \frac{a}{2} = \frac{B}{2}. \quad (6)$$

Now, if we equate the complex part of (5b) with the complex part of (4), we have that

$$\frac{a}{2} = \frac{B^2}{4} + \frac{1}{4}. \quad (7)$$

If we add Eqs. (6) and (7), we get

$$x^2 = \frac{B}{2} + \frac{B^2}{4} + \frac{1}{4} = \left( \frac{B}{2} + \frac{1}{2} \right)^2. \quad (8)$$

Form this we have that

$$x = \pm \left( \frac{B}{2} + \frac{1}{2} \right). \quad (9)$$

But from (7), we can solve for  $B$  as a function of  $a$ :

$$B = \pm \sqrt{2a - 1}. \quad (10)$$

Hence, tentatively,

$$x = \pm \left( \frac{\pm \sqrt{2a - 1}}{2} + \frac{1}{2} \right) = \pm \frac{1}{2} (1 \pm \sqrt{2a - 1}). \quad (11)$$

However,  $x$  cannot have four roots. But since we're looking for real roots, the  $\sqrt{a - x^2} > 0$  in (1), forcing  $x$  to be positive. Therefore,

$$x = \frac{1}{2} (1 \pm \sqrt{2a - 1}). \quad (12)$$