Math Diversion 1008

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The best theory is inspired by practice.
— Donald Knuth

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Source: https://www.youtube.com/watch?v=SELJwouvdSI
Title: A Radical Adventure
 Presenter: SyberMath
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1 Problem

Given the relations

$$\sqrt{x} + \sqrt{y} = 3\,, \tag{1a}$$
$$\sqrt{x+5} + \sqrt{y+3} = 5\,, \tag{1b}$$

find the positive real solutions for x, y.

2 Solution

We need to define a couple helper variables

$$A \equiv \sqrt{x} - \sqrt{y}\,, \tag{2a}$$
$$A \equiv \sqrt{x+5} - \sqrt{y+3}\,. \tag{2b}$$

Now,

$$(\sqrt{x} + \sqrt{y})((\sqrt{x} - \sqrt{y}\,) = x - y\,, \tag{3}$$

but using (1a) and (2a), we have that

$$3A = x - y\,. \tag{4}$$

From this we get

$$A = \frac{x-y}{3}\,. \tag{5}$$

Next, we're going to employ (1a) and (2a) to get a different relation on A:

$$(\sqrt{x} + \sqrt{y})^2 = 9\,, \tag{6a}$$
$$(\sqrt{x} - \sqrt{y})^2 = A^2\,, \tag{6b}$$

which becomes

$$x + 2\sqrt{xy} + y = 9\,, \tag{7a}$$
$$x - 2\sqrt{xy} + y = A^2\,. \tag{7b}$$

On adding these together, the square root terms will cancel, and then on solving for A^2, we get

$$A^2 = 2x + 2y - 9\,. \tag{8}$$

Then, on eliminating A between this last equation and (5), we have that

$$\frac{(x-y)^2}{9} = 2(x+y) - 9\,. \tag{9}$$

On performing a similar set of steps on (1b) and (2b) as we just did, we get the relation

$$2x + 2y - 9 = \frac{(x-y+2)^2}{25}\,. \tag{10}$$

Using WolframAlpha to solve (9) and (9), I get for solution to (x, y):

$$\{(4,1),\ (\frac{121}{64}, \frac{169}{64})\}\,. \tag{11}$$