Math Diversion 1011

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Lifelong learning is no longer a luxury but
a necessity for employment.
— Jay Samit

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Source: https://www.youtube.com/watch?v=j45XT7h7Yss
Title: Fun Math Problem
 Presenter: Andy Math
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1 Problem

Given the relation

$$40^{x-1} = 2^{2x+1}\,, \tag{1}$$

find real solutions for x.

2 Solution

First, we take the logarithm base 2, to get

$$(x-1)\log_2 40 = 2x+1\,, \tag{2}$$

Now,

$$\log_2 40 = \log_2(2^3\cdot 5) = \log_2(2^3) + \log_2 5 = 3+\alpha\,, \tag{3}$$

where $\alpha \equiv \log_2 5$ Thus, (2) becomes

$$(x-1)(3+\alpha) = 2x+1\,. \tag{4}$$

On taking a big step, we have that

$$(1+\alpha)x = 1+(3+\alpha)\,. \tag{5}$$

Solving for x, we get

$$x = \frac{4+\alpha}{1+\alpha} = \frac{4+\log_2 5}{1+\log_2 5}\,. \tag{6}$$

Using that $\log_2 5 = (\log 5)/(\log 2)$, this becomes

$$x = \frac{4\log 2+\log 5}{\log 2+\log 5}\,. \tag{7}$$