Math Diversion 1014

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> It is customary, in developing the method of Green's functions,
> to require that the boundary conditions be homogeneous,
> with all of the inhomogeneity contained
> contained in the differential equation.
> — Michael D. Greenberg
> *Applications of Green's Functions*
> *in Science and Engineering* (p.5)

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Source: https://www.youtube.com/watch?v=3VQ_oHa07oo
Title:  Can YOU Find the Radius of This Circle?
 Presenter: Math Queen
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1 Problem

Find the radius r of the circle inscribed in the right trapezoid:

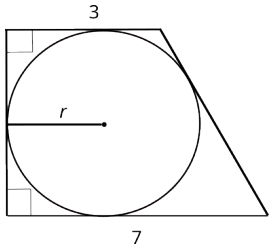

Figure 1. The basic setup.

2 Solution

I think that I can add more symmetry to the problem by inserting the right trapezoid into a right triangle, as such

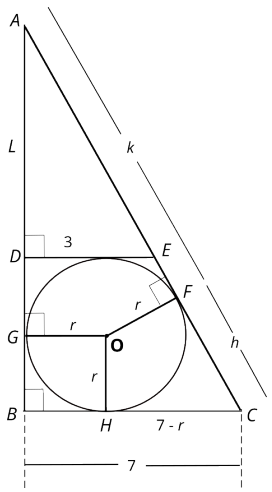

Figure 2. I can now solve this problem using similar triangles, congruent triangles, and the Pythagorean Theorem.

Let's start with similar triangles: $\triangle ADE \sim \triangle ABC$, from which we get the proportion:

$$\frac{L}{3} = \frac{L+2r}{7}\,. \tag{1}$$

We can solve this for L:

$$L = \frac{3}{2}r\,. \tag{2}$$

Let's turn now to the two congruent triangles: $\triangle AOG$ and $\triangle AOF$. (I used SSA to prove this.) From this congruence, we have that

$$k = L + r = \frac{5}{2}r\,. \tag{3}$$

By similar reasoning, we get that: $\triangle FOC \cong \triangle HOC$. (I used SSA to prove this.) From this congruence, we have that

$$h = 7 - r\,. \tag{4}$$

And now for the Pythagorean Theorem: From right triangle $\triangle ABC$, we get

$$(L+2r)^2 + (7)^2 = (k+h)^2\,. \tag{5}$$

So, when I used all these equations together and simplified, I got

$$10r^2 = 21r\,. \tag{6}$$

Solving for r, I got

$$r = \frac{21}{10}\,. \tag{7}$$