Math Diversion 1019

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The main problem of topology is to find useful,
necessary and sufficient conditions, other than
just the definition, for two spaces
to be homeomorphic.
—John B. Freleigh
A First Course in
Abstract Algebra

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Source: https://www.youtube.com/watch?v=0q_933QGmWM
Title: Complex Analysis is Destroying this...
 Presenter: Dr PK Math
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1 Problem

Given the relation

$$z^{\frac{1+i}{\sqrt{2}}} = \left(\frac{1-i}{\sqrt{2}}\right)^z \,, \tag{1}$$

solve for the complex number z but add no unnecessary indexed solutions.

2 Solution

First, we note that

$$\frac{1+i}{\sqrt{2}} = e^{i\pi/4} \quad \text{and} \quad \frac{1-i}{\sqrt{2}} = e^{-i\pi/4} \,, \tag{2}$$

Thus (1) becomes

$$z^{e^{i\pi/4}} = \left(e^{-i\pi/4}\right)^z \,. \tag{3}$$

Now, we take the natural logarithm across this equation:

$$e^{i\pi/4}\ln z = z\ln e^{-i\pi/4} = -(i\pi/4)z \,. \tag{4}$$

Applying a little algebra, we get

$$-z^{-1}\ln z = e^{-i\pi/4}(i\pi/4) \,, \tag{5}$$

or rather

$$z^{-1}\ln z^{-1} = e^{-i\pi/4}(i\pi/4)\,. \tag{6}$$

Now, using the lemma from the theory of the Lambert W function, that

$$W(y\ln y) = \ln y\,, \tag{7}$$

then

$$\ln z^{-1} = W(\tfrac{1}{4}i\pi e^{-i\pi/4})\,. \tag{8}$$

After exponentiating, we have that

$$z^{-1} = e^{W(\frac{1}{4}i\pi e^{-i\pi/4})}\,. \tag{9}$$

And finally,

$$z = e^{-W(\frac{1}{4}i\pi e^{-i\pi/4})}\,. \tag{10}$$

So, yes, I could have factored in $e^{2\pi in}$ early on, where $n \in \mathbb{Z}$, but I'll leave that for the interested reader to do.