Math Diversion 1022

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Science has always prided itself on being empirical
and believing only what could be verified.
— Bertrand Russell
The Basic Writings of Bertrand Russell.

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Source: https://tutorial.math.lamar.edu/Classes/CalcI/LogDiff.aspx
Title: Induction
 Presenter: Patrick
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1 Problem

Using induction, prove the relation

$$D_x x^n = n x^{n-1} \,. \tag{1}$$

2 Solution

This is not so much an exercise in proving the relation as it is an exercise in using induction as the main mode of proof. It's a practice problem in learning induction.

Now, to use induction, we begin by proving the base case, which in this problem is for $n = 1$:

$$D_x x = 1 \cdot x^0 = 1 \,, \tag{2}$$

so that works out.

Next, we assume the truth of the kth case (for $k > 1$ but otherwise arbitrary). In other words, we assert that

$$D_x x^k = k x^{k-1} \,, \tag{3}$$

which is referred to as the Inductive Hypothesis. What we need to show is, given both (2) and (3), that

$$D_x x^{k+1} = (k+1) x^k \,. \tag{4}$$

Okay, let's begin with the LHS of (5):

$$\begin{aligned} D_x x^{k+1} &= D_x(x^k \cdot x) \\ &= (D_x x^k) \cdot x + x^k \cdot (D_x x) \quad \text{(by Product Rule)} \\ &= kx^{k-1} \cdot x + x^k(1) \quad \text{(by (3))} \\ &= kx^k + x^k \\ &= (k+1)x^k \,. \end{aligned} \tag{5}$$

Technically, we've done what we needed to do, but if you want to present it with a bow, you can replace k by $n-1$, to get

$$D_x x^n = nx^{n-1} \,. \tag{6}$$