

Math Diversion 1023

P. Reany

January 25, 2026

The inquiring scientist approaches each problem with an open mind and with the highest type of intellectual integrity, willing and eager to follow wherever the facts may lead. Whenever possible, he analyzes a complex problem into its simplest components and devises carefully controlled experiments which clearly reveal cause and effect relationships. He is critical of the validity of his own data and subjects his hypotheses to exhaustive experimental tests before arriving at even tentative conclusions. In all his work, he relies upon a faith that natural phenomena are reproducible and that the universe is orderly.

—Sisler, H.H., C.A. VanderWerf & A.W. Davidson.
College Chemistry — A Systematic Approach.

Source: <https://tutorial.math.lamar.edu/Classes/CalcI/LogDiff.aspx>

Title: Logarithmic Differentiation

Presenter: Patrick

1 Problem

Evaluate the derivative

$$D_x x^x = ?, \tag{1}$$

where $x \neq 0$.

2 Solution

If you know my style, you know that I like to give funny expressions names. So, let

$$f(x) \equiv x^x. \tag{2}$$

Now, if we take the derivative of $f(x)$ by x that is what we are asked to find. But we're going to accomplish this the sneaky way.

First, we apply the natural logarithm across (2) to arrive at¹

$$\ln f(x) = x \ln x. \tag{3}$$

¹When we take a logarithm of a function before we differentiate it, we call this procedure *logarithmic differentiation*.

Now we differentiate across this equation to get

$$\frac{f'(x)}{f(x)} = \ln x + 1. \quad (4)$$

Next, we just multiply through by $f(x)$ and use (2), to get

$$D_x f(x) = f'(x) = x^x(\ln x + 1). \quad (5)$$