Math Diversion 1024

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> Science is a quest for simplicity.
> — Atkins, P.W. & Jones, L.
> *Chemical Principles,*
> *the Quest for Insight*

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Source: https://www.youtube.com/watch?v=hMhds8tMZxk
Title: Maths Olympiad | Only 10%
 Presenter: Rashel's Classroom
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1 Problem

Given the relations

$$xy = 5\,, \tag{1a}$$
$$yz = 10\,, \tag{1b}$$
$$zx = 15\,, \tag{1c}$$

find

$$\phi = x + y + z\,. \tag{2}$$

2 Solution

We begin by dividing (1a) by (1b) to get

$$\frac{x}{z} = \frac{1}{2}\,. \tag{3}$$

Now, if we multiply this last equation by (1c)

$$x^2 = \frac{15}{2}\,, \tag{4}$$

from which we get that

$$x = \sqrt{\frac{15}{2}} = \sqrt{\frac{45}{6}}\,, \tag{5}$$

We can now solve for y and z:

$$y = \frac{5}{x} = \sqrt{\frac{10}{3}} = \sqrt{\frac{20}{6}}\,, \tag{6a}$$

$$z = \frac{15}{x} = \sqrt{\frac{2 \cdot 15 \cdot 6}{6}}\,. \tag{6b}$$

Therefore,

$$\phi = \sqrt{\frac{45}{6}} + \sqrt{\frac{20}{6}} + \sqrt{\frac{2 \cdot 15 \cdot 6}{6}} = 11\sqrt{\frac{5}{6}}\,. \tag{7}$$