

# Math Diversion 1030

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Beware of the natural tendency to be self-sufficient.  
In the good times it makes us arrogant;  
in the bad times, fearful.  
— a warning

Source: <https://www.youtube.com/watch?v=7Dby0KR7byc>  
Title: A Weird Log Equation | Can You Solve?  
Presenter: SyberMath

## 1 Problem

Given the relation

$$\log_3(x - 2) = \log_5 x, \quad (1)$$

solve for  $x$ .

## 2 Solution

Judging by the nature of the two functions involved, their graphs should intersect at most in a single point.

Now, by inspection, you might be able to solve the relation to get  $x = 5$ , which is correct.

But an equation without logarithms might be easier to solve. Let's begin by replacing two bases by just one, base 10:

$$\frac{\log(x - 2)}{\log 3} = \frac{\log x}{\log 5}. \quad (2)$$

Now, let

$$\alpha = \frac{\log 3}{\log 5} = \log_5 3, \quad (3)$$

then (2) becomes

$$\log(x - 2) = \alpha \log x, \quad (4)$$

or rather

$$\log(x - 2) = \log x^\alpha. \quad (5)$$

On raising 10 to this last equation, we have that

$$x - 2 = x^{\log_5 3}. \quad (6)$$

Well, we still have a logarithm we can get rid of by letting

$$x = 5^y. \quad (7)$$

On using this in (6), we have

$$5^y - 2 = (5^y)^{\log_5 3} = 5^{\log_5 3^y} = 3^y, \quad (8)$$

or

$$5^y - 2 = 3^y, \quad (9)$$

where, by inspection, we see that  $y = 1$ . And if  $y = 1$ , then

$$x = 5. \quad (10)$$