Math Diversion 1031

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> You either get control of your lusts and feelings of entitlement, or they will get control of you.
> — The Author

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 Source: https://www.youtube.com/watch?v=psA6ZKKkPcQ
Title: Cambridge Maths Entrance Exam Question
 Presenter: Math Beast
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1 Problem

Given the relation

$$(-19)^z = 19\,, \tag{1}$$

solve for z.

2 Solution

We begin by taking a natural logarithm

$$z \ln(-19) = \ln 19\,. \tag{2}$$

But this is the age of complex numbers, so we must broaden what we mean by -1.

$$-1 = e^{i\pi} e^{2i\pi k} = e^{i\pi + 2i\pi k} \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{3}$$

So, (2) becomes

$$z \ln(19 e^{i\pi + 2i\pi k}) = \ln 19 \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{4}$$

But

$$\ln(19 e^{i\pi + 2i\pi k}) = \ln 19 + i\pi + 2i\pi k = \ln 19 + i\pi(1 + 2k)\,. \tag{5}$$

Thus,

$$z(\ln 19 + i\pi(1 + 2k)) = \ln 19 \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{6}$$

And finally

$$z = \frac{\ln 19}{\ln 19 + i\pi(2k+1)} \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{7}$$

Note: There are various ways to insert an 'exponential unity' ($1 = e^{2\pi in}$) into an equation. WolframAlpha did it differently and got a different answer.