Math Diversion 1036

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More than a century since its debut, representation theory
has served as a key ingredient in many of the most important
discoveries in mathematics. Yet its usefulness
is still hard to perceive at first.
— Kevin Hartnett

(Representation theory — among other uses, it is the representation of elements of an arbitrary group by the elements of a linear map on a vector space. Once a basis is chosen, the linear map can take the form of a matrix group.)

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Source: https://www.youtube.com/watch?v=dyicSIJAJ2s
Title: Why Most People Can't Find All 3 Solutions
 Presenter: Mental Math
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1 Problem

Given the relation

$$x^2 = 2^{-x}\,, \tag{1}$$

find three real values of x.

2 Solution

This looks like a job for the Lambert W function. (See my write-up on it for assistance.)

But before we get to that, we could by inspection see that one solution is $x = -2$.

Anyway, let's reform (1) to

$$x = \pm\frac{1}{\sqrt{2}^{\,x}}\,, \tag{2}$$

but this can be rewritten as

$$x\sqrt{2}^{\,x} = \pm 1\,. \tag{3}$$

Next, we take the Lambert W function base $\sqrt{2}$, to get

$$x = W_{(\sqrt{2})}(\pm 1) = \frac{W_n(\pm \ln \sqrt{2})}{\ln \sqrt{2}} = \frac{W_n(\pm \frac{1}{2} \ln 2)}{\ln \sqrt{2}} \quad \text{for } n \in \mathbb{Z}\,. \tag{4}$$

There are three real values of W. We start with the plus sign:

$$x_0^+ = \frac{W_0(+\frac{1}{2} \ln 2)}{\ln \sqrt{2}} = 0.766665\,. \tag{5}$$

Next, we go with the minus sign:

$$x_0^- = \frac{W_0(-\frac{1}{2} \ln 2)}{\ln \sqrt{2}} = -2\,, \tag{6}$$

$$x_{-1}^- = \frac{W_{-1}(-\frac{1}{2} \ln 2)}{\ln \sqrt{2}} = -4\,. \tag{7}$$

By the way, I calculated these values musing WolframAlpha and the 'ProductLog' command.