Math Diversion 1037

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> Chop your own wood and it will warm you twice.
> — Henry Ford

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Source: https://www.youtube.com/watch?v=C4oTVRdTlmo
Title: Nice Math Olympiad Algebra Simplification
 Presenter: Math Beast
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1 Problem

Given the relation

$$x\sqrt{x} = x + \sqrt{x}\,, \tag{1}$$

find the real values of $x \geq 0$.

2 Solution

First, it's obvious that one solution is $x = 0$.

The next thing to do is to convert to a form that less intimidating. So, let

$$x = u^2\,. \tag{2}$$

Then (1) becomes

$$u^3 = u^2 + u\,. \tag{3}$$

We already know that $x = 0$ is a solution, hence $u = 0$ is a solution that we don't care to include in (3). So, let's divide through by u, to get

$$u^2 = u + 1\,. \tag{4}$$

This equation has solutions

$$u = \frac{1 \pm \sqrt{5}}{2}\,. \tag{5}$$

But since $x = u^2$, then, on using the last two equations:

$$x = u^2 = \frac{1 \pm \sqrt{5}}{2} + 1 = \frac{3 \pm \sqrt{5}}{2}\,. \tag{6}$$

Note: WolframAlpha discarded the negative-sign root, even though it's greater than zero. The graph says it all.