

Math Diversion 1038

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First things first...But not necessarily in that order.

— Doctor Who

Source: <https://www.youtube.com/watch?v=PgEPrEbru5M>

Title: solving a nested logarithmic equation

Presenter: blackpenredpen

1 Problem

Given the relation

$$\log(x + \ln x) = 1, \tag{1}$$

find the real values of x . (Note: $\log y = \log_{10} y$.)

2 Solution

First, we raise 10 to the given equation, yielding

$$x + \ln x = 10^1 = 10. \tag{2}$$

Next, we raise e to the last equation:

$$e^{x+\ln x} = e^{10}. \tag{3}$$

But

$$e^{x+\ln x} = e^x e^{\ln x} = x e^x; \tag{4}$$

hence (3) becomes

$$x e^x = e^{10}. \tag{5}$$

So, on taking the Lambert W function across this equation, we have that

$$x = W_n(e^{10}). \tag{6}$$

You can use WolframAlpha to find numeric values for x .