Math Diversion 1039

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February 12, 2026

People often overlook the obvious.
— Doctor Who

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Source: https://www.youtube.com/watch?v=q0wnL9roqTI
Title: An Absolutely Complex Equation | P606
 Presenter: aplusbi
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1 Problem

Given the relation

$$|z|i - z = 1 + 2i \,, \tag{1}$$

find the values of z.

2 Stuff to Know

If we let $z = a+bi$ and $\overline{z} = a-bi$, then taking into account complex conjugation,

$$\overline{i} = -i \,, \tag{2a}$$

$$z + \overline{z} = 2a \,, \tag{2b}$$

$$z - \overline{z} = 2bi \,, \tag{2c}$$

$$|z|^2 = z\overline{z} = a^2 + b^2 \,. \tag{2d}$$

3 Solution

First, we conjugate the given equation, yielding

$$-|z|i - \overline{z} = 1 - 2i \,, \tag{3}$$

Next, on adding together (1) and (3), we get

$$-2a = 2 \,, \tag{4}$$

yielding

$$a = -1 \,. \tag{5}$$

On subtracting (3) from (1), we get

$$|z| - b = 2\,, \tag{6}$$

which becomes (on solving for $|z|$ and squaring and using (2d)),

$$a^2 + b^2 = b^2 + 4b + 4\,, \tag{7}$$

which simplifies down to

$$a^2 = 4b + 4\,. \tag{8}$$

Using this equation and (5), we solve for b, getting

$$b = -3/4\,. \tag{9}$$

Hence, the solution to the Given equation is

$$z = -1 - \frac{3i}{4}\,. \tag{10}$$