Math Diversion 1042

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'Cause if love won't fly on its own free will,
it's gonna catch that outbound plane.
— Suzy Bogguss, "Outbound Plane"

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Source: https://www.youtube.com/watch?v=izGAhb2YgE8
Title: The Prettiest Radical Identity
 Presenter: SyberMath
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1 Problem

Given the expression

$$\phi = \sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}}\,, \tag{1}$$

simplify ϕ.

Note: Clearly, $\phi > 0$. Why?

2 Solution

Start by squaring!

$$\begin{aligned}\phi^2 &= (2+\sqrt{3}) - 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} + (2-\sqrt{3}) \\ &= 4 - 2\sqrt{4-3} \\ &= 4 - 2 \\ &= 2\,.\end{aligned}$$

Taking the square root across this equation, we get

$$\phi = \sqrt{2}\,. \tag{2}$$

This is one of those rare instances where squaring a complicated expression simplifies rather than further complicates.