Math Diversion 1045

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In the middle of difficulty lies opportunity.
— John Archibald Wheeler

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1 Problem

While hiking up and then down a trail, Rolf spent 60% of his time hiking uphill and 40% hiking back down. If he averaged 2 mph uphill, what was his average speed round trip?

2 Solution

Let $\overline{v}$ stand for average speed and T stand for the total time of the hike.

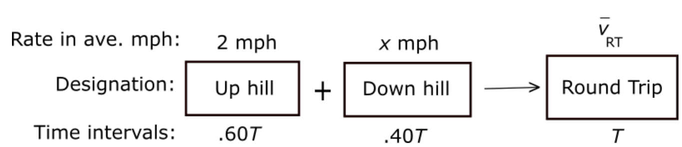

Figure 3. Rolf's hike parsed as totals and parts.

In the above figure, I used x for the average speed going down, but now I'll use $\overline{v}_{\downarrow}$. If we balance on the total distance across the process, we get

$$\overline{v}_{\uparrow}(.60T) + \overline{v}_{\downarrow}(.40T) = \overline{v}_{\rm RT}T \tag{1}$$

where 'RT' is short for 'round trip'. Solving for $\overline{v}_{\rm RT}$, we get

$$\overline{v}_{\rm RT} = \overline{v}_{\uparrow}(.60) + \overline{v}_{\downarrow}(.40)\,. \tag{2}$$

We were given two other pieces of information that we need right now. First, that $\overline{v}_\uparrow = 2\,\text{mph}$, and second, that the uphill distance is the same as the downhill distance: $D_\uparrow = D_\downarrow$, giving us the constitutive relation

$$\overline{v}_\uparrow(.60T) = \overline{v}_\downarrow(.40T)\,, \tag{3}$$

from which we get that

$$\overline{v}_\downarrow = \frac{3}{2}\,\overline{v}_\uparrow = 3[\text{mph}]\,. \tag{4}$$

Substituting this information into (2) gives

$$\overline{v}_{\text{RT}} = 2[\text{mph}](.60) + 3[\text{mph}](.40) = 2.4[\text{mph}]\,. \tag{5}$$