Math Diversion 1046

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Unless you try to do something beyond what you have
already mastered, you will never grow.
— Ralph Waldo Emerson

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Source: https://www.youtube.com/watch?v=OIK7N5Odn_k
Title: 98% Failed To Solve This Math
 Presenter: Math Beast
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1 Problem

Given the relation

$$f(f(x)) = x^2 - x + 1\,, \tag{1}$$

find the value of $f(0)$.

2 Solution

First, let's introduce a shorthand name for $f(0)$,

$$k \equiv f(0)\,. \tag{2}$$

Then

$$f(f(0)) = f(k) = 0^2 - 0 + 1 = 1\,. \tag{3}$$

We can for future use immediately write down

$$f(f(k)) = k^2 - k + 1\,, \tag{4}$$

$$f(f(1)) = 1^2 - 1 + 1 = 1\,, \tag{5}$$

so, from (3), we get that

$$f(f(k)) = f(1)\,. \tag{6}$$

Now, let

$$f(1) \equiv a\,, \tag{7}$$

then

$$f(f(1)) = f(a) = 1\,, \tag{8}$$

where we used (5). Hence,

$$f(f(a)) = f(1)\,, \tag{9}$$

giving us,

$$a^2 - a + 1 = a\,, \tag{10}$$

where we used (1) and (7). This last equation becomes

$$(a-1)^2 = 0\,, \tag{11}$$

which gives us

$$a = 1\,. \tag{12}$$

Applying this result to (7), we have that

$$f(1) = 1\,. \tag{13}$$

On using this result in (6), we get

$$f(f(k)) = 1\,. \tag{14}$$

Using the relation (4), we have that

$$k^2 - k + 1 = 1\,, \tag{15}$$

which has solutions

$$k = 1 \quad \text{and} \quad k = 0\,, \tag{16}$$

Using this first value in (3), we get

$$f(1) = 1\,, \tag{17}$$

which we already knew (consistency check). Using the second value in it, we get what we needed to show

$$f(0) = 1\,. \tag{18}$$