Math Diversion 1047

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> You either get control of your lusts and feelings of entitlement, or they will get control of you.
> — The Author

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Source: https://www.youtube.com/watch?v=_jxh0eTwsQA
Title: A Wonderful Math Olympiad Algebraic Question
 Presenter: Mathematics & Statistics Guru
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1 Problem

Given the relation

$$\sqrt{2}^{\sqrt{x}} - \sqrt{2}^{\sqrt{y}} = 112\,, \tag{1}$$

solve for x, y where $x, y \in \mathbb{R}^+$.

2 Solution

Let's begin with the substitution:

$$\sqrt{x} = 2u \quad \text{and} \quad \sqrt{y} = 2v\,. \tag{2}$$

Then (1) becomes

$$2^u - 2^v = 112\,. \tag{3}$$

Making the assumption that u, v are integers, what are the likely powers of 2 that we can try as differences? Well, u, v can't be too large or too small:

$$2^3 = 8,\ 2^4 = 16,\ 2^5 = 32,\ 2^6 = 64,\ 2^7 = 128,\ 2^8 = 256,\ 2^9 = 512\ . \tag{4}$$

Trial and error gives us

$$128 - 16 = 112\,. \tag{5}$$

Hence $u = 7$ and $v = 4$. Therefore

$$x = (2u)^2 = 196 \quad \text{and} \quad v = (2v)^2 = 64\,. \tag{6}$$