Math Diversion 1048

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> Since the mathematicians have invaded the theory of
> relativity, I do not understand it myself anymore.
> — Albert Einstein

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Source: The Ether of Great Mathematical Ideas
Title: An alpha-transformation problem
 Presenter: Patrick
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1 Problem

Given the relation

$$t^t = 4^{t+16}\,, \tag{1}$$

solve for t where $t \in \mathbb{R}^+$.

2 Solution

Let's begin with the substitution:

$$t = 4^\alpha\,. \tag{2}$$

Then (1) becomes

$$4^{\alpha 4^\alpha} = 4^{4^\alpha + 4^2}\,. \tag{3}$$

On equating exponents, we have that

$$\alpha 4^\alpha = 4^\alpha + 4^2\,. \tag{4}$$

After a little algebra, we get

$$(\alpha - 1)4^{\alpha - 2} = 1\,. \tag{5}$$

By inspection, we have that

$$\alpha = 2\,. \tag{6}$$

Hence,

$$t = 16\,. \tag{7}$$