Math Diversion 1049

P. Reany

March 3, 2026

It is a capital mistake to theorize
in advance of the facts.
— Sherlock Holmes (Jeremy Brett)
[Episode *The Second Stain*]

```
Source: https://www.youtube.com/watch?v=ETzbGnYA5vI
Title:  Cambridge Maths Exam Question
 Presenter: Math Beast
```

1 Problem

Given the relation

$$x^{\log 2x} = 5 \quad \text{(the logarithm is base 10)}\,, \tag{1}$$

solve for x where $x \in \mathbb{R}$.

Note: It will be handy to remember that $\log 10 = 1$.

2 Solution

Hint: Let's not overlook finding a solution by inspection.

First, we take the logarithm across (1), to get

$$(\log 2x)(\log x) = \log 5\,. \tag{2}$$

By inspection, we can see that $x = 5$ is a solution. But is there another? We should suspect that there is because we can write this last equation as a quadratic in $\log x$.

But instead of solving for this quadratic, we can perhaps find the second solution by using a simple algebraic identity and the properties of logarithms. So, The algebraic identity gives us:

$$[-(\log 2x)][-(\log x)] = \log 5\,, \tag{3}$$

which becomes

$$\big(\log(2x)^{-1}\big)\big(\log(x)^{-1}\big) = \log 5\,, \tag{4}$$

or more simply as

$$\big(\log\frac{1}{2x}\big)\big(\log\frac{1}{x}\big) = \log 5\,. \tag{5}$$

Once more using inspection, I get $x = 1/10$. And that makes two solutions. Thus, the two real solutions are

$$x = 5, 1/10\,. \tag{6}$$