

Math Diversion 1052

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Every good mathematician is at least half a philosopher,
and every good philosopher is at least
half a mathematician.
— Gottlob Frege

Source: <https://www.youtube.com/watch?v=kDLP31Zh2TI>

Title: ALGEBRA CHALLENGE

Presenter: Maths Simplified Solutions

1 Problem

Given the relation

$$x^{332} + x^{-332} = 963, \quad (1)$$

find the real values of

$$\phi = x^{166} - x^{-166}. \quad (2)$$

2 Solution

Let's solve this problem by use of the unipodal algebra.

Let

$$a = x^{166}u_+ + x^{-166}u_- \quad (3a)$$

$$= \frac{1}{2}(x^{166} + x^{-166}) + \frac{1}{2}(x^{166} - x^{-166})u \quad (3b)$$

$$= \frac{1}{2}(x^{166} + x^{-166}) + \frac{1}{2}\phi u, \quad (3c)$$

where we use (2).

On squaring (3a), we get

$$a^2 = x^{332}u_+ + x^{-332}u_- . \quad (4)$$

Expressed in standard form, this become

$$a^2 = \frac{1}{2}(x^{332} + x^{-332}) + \frac{1}{2}(x^{332} + x^{-332})u \quad (5)$$

$$= \frac{1}{2}(963) + \frac{1}{2}(x^{332} + x^{-332})u . \quad (6)$$

Now, on squaring (3c), we have that

$$a^2 = \frac{1}{4}(x^{166} + x^{-166})^2 + \frac{1}{4}\phi^2 + \frac{1}{2}(x^{166} + x^{-166})\phi u. \quad (7)$$

But,

$$(x^{166} + x^{-166})^2 = (x^{332} + x^{-332} + 2) = 965, \quad (8)$$

where we used (1). So, on using this last result in (6), we get

$$a^2 = \frac{1}{4}(965) + \frac{1}{4}\phi^2 + \frac{1}{2}(x^{166} + x^{-166})\phi u. \quad (9)$$

On equating the scalar part of this last equation with the scalar part of (6), we get

$$\frac{1}{2}(963) = \frac{1}{4}(965) + \frac{1}{4}\phi^2, \quad (10)$$

which we can easily solve for ϕ , getting

$$\phi = \pm 31. \quad (11)$$