

Math Diversion 1056

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Average talent, plus hard work and dedication,
will always beat talent by itself.
— Clinton Anderson

Source: <https://www.youtube.com/watch?v=yCmtrra0qZI>
Title: Solve for X
Presenter: PreMath

1 Problem

Given the relation

$$3125^x = x^{-1}, \quad (1)$$

solve for real values of x . (Calculators allowed for basic arithmetic.)

2 Solution

I choose to solve this problem using the Lambert W function. First, we put the Given relation into the following form:

$$x3125^x = 1. \quad (2)$$

Now we take the Lambert W function base 3125 across this equation:

$$x = W_{(3125)}(1) = \frac{W_0(1 \cdot \ln 3125)}{\ln 3125}. \quad (3)$$

But $3125 = 5^5$, hence

$$x = \frac{W_0(5 \ln 5)}{5 \ln 5} = \frac{\ln 5}{5 \ln 5} = \frac{1}{5}. \quad (4)$$

Note: $x = \frac{W_0(\ln 3125)}{\ln 3125}$ is a perfectly correct answer. It's only because the base 3125 is of the form a^a that we could 'extricate' the answer out of the Lambert W function by use of the identity:

$$W(a \ln a) = \ln a \quad \text{for } a > 0. \quad (5)$$