Math Diversion 1060

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> You have to know what to look for, so you can spot it.
> — Papago drug-enforcement
> border scout

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Source:  https://www.youtube.com/watch?v=60G2F3I9zM8
Title: Power Tower | Can you solve for X?
 Presenter: PreMath
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1 Problem

Given the relation

$$x^{x^x} = (0.5)^{\sqrt{2}}\,, \tag{1}$$

solve for real values of x.

I choose to rewrite the Given as

$$x^{x^x} = (2)^{-\sqrt{2}}\,. \tag{2}$$

2 Solution

We begin by applying the logarithm base 2 to (2):

$$x^x \log_2 x = -\sqrt{2}\,. \tag{3}$$

Next, we'll try an α transformation, given by

$$x = 2^{-\alpha}\,. \tag{4}$$

On substituting this into (3), we have that

$$-\alpha 2^{-\alpha 2^{-\alpha}} = -\sqrt{2}\,, \tag{5}$$

or put more simply,

$$\alpha 2^{-\alpha 2^{-\alpha}} = 2^{1/2}\,, \tag{6}$$

We again apply the logarithm base 2:

$$\log_2 \alpha + \log_2 2^{-\alpha 2^{-\alpha}} = 1/2\,, \tag{7}$$

or more simply

$$\log_2 \alpha - \alpha 2^{-\alpha} = 1/2\,. \tag{8}$$

By inspection, I suspect that α is either 1 or 2. On trying them, I find that $\alpha = 2$. Thus, from (4), we get

$$x = 2^{-2} = 1/4\,. \tag{9}$$