Math Diversion 1061

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March 25, 2026

Solving math puzzles can enhance cognitive functions such
as problem-solving, logical reasoning, and memory,
contributing to overall brain health.
— Daniel Levitin (renowned neuroscientist)

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Source:  https://www.youtube.com/watch?v=1b1G2hARThc
Title: Oxford Entrance Question
 Presenter: PreMath
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1 Problem

Given the relation

$$2^{x+2} + 4^x = 4 + 8^x \,, \tag{1}$$

solve for real values of x.

2 Solution

Let rewrite the Given equation to the following form:

$$4 \cdot 2^x + 2^{2x} = 4 + 2^{3x} \,, \tag{2}$$

and then one more rewrite:

$$(2^x)^3 - (2^x)^2 - 4(2^x) + 4 = 0 \,. \tag{3}$$

On setting

$$y = 2^x \,, \tag{4}$$

we get

$$y^3 - y^2 - 4y + 4 = 0 \,. \tag{5}$$

As I expect these these polynomials have at least one "small" integer solution, I tried $y = 1$, which worked, giving us the factored form:

$$(y - 1)(y^2 - 4) = 0 \,, \tag{6}$$

where I used long division to get the factor $y^2 - 4$.

Thus, we have three possible values for y:

$$y = 1, 2, -2\,, \tag{7}$$

which correspond to three possible values for 2^x:

$$2^x = 1, 2, -2\,, \tag{8}$$

which correspond to two possible real values for x:[1]

$$x = 0, 1\,. \tag{9}$$

[1]For real x, 2^x is never negative.