Math Diversion 1063

P. Reany

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If others would think about mathematical truths as
deeply and as continuously as I have, they
would make my discoveries.
— Carl Friedrich Gauss

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Source:  The Ether of Great Mathematical Ideas
Title: DIY Problem
 Presenter: Patrick
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1 Problem

Given the relation

$$6^x - 9x = 2\,, \tag{1}$$

solve for all values of x.

2 Solution

This looks like a job for the Lambert W function!

But we have to put the Given into a usable form first. So, we rewrite it as

$$6^x = 9x + 2 = 9(x + \tfrac{2}{9})\,, \tag{2}$$

then set $u = x + \frac{2}{9}$, or

$$x = u - \tfrac{2}{9}\,. \tag{3}$$

On substituting these into (2), we have that

$$6^{u-2/9} = 9u\,. \tag{4}$$

After a bit of algebra, this becomes

$$-u6^{-u} = -\frac{1}{9 \cdot 6^{2/9}}\,. \tag{5}$$

At this point, we can take the Lambert W function base 6 across this last equation,[1] to get

$$-u = W_{(6)}(-\tfrac{1}{9\cdot 6^{2/9}}) = \frac{W_n(-\frac{1}{9\cdot 6^{2/9}}\cdot \ln 6)}{\ln 6}\,, \tag{6}$$

where n is an integer. Using (3), we have that

$$-x - \tfrac{2}{9} = \frac{W_n(-\frac{1}{9\cdot 6^{2/9}}\cdot \ln 6)}{\ln 6}\,, \tag{7}$$

which gives us

$$x = -\frac{W_n(-\frac{1}{9\cdot 6^{2/9}}\cdot \ln 6)}{\ln 6} - \frac{2}{9}\,. \tag{8}$$

If you are interested in real values of this last equation, try $n = 0, -1$ in WolframAlpha, using ProductLog[].

[1]See my write-up on the Lambert W function to see what I mean by applying various 'bases' to the Lambert W function.