

Math Diversion 1064

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March 29, 2026

I take the positivist viewpoint that a physical theory is just a mathematical model and that it is meaningless to ask whether it corresponds to reality. All that one can ask is that its predictions should be in agreement with observation. — Stephen Hawking
[*The Nature of Space and Time*, p.3-4]

Source: https://www.youtube.com/watch?v=gIYT9Vp_gdk
Title: This Radical Equation Has a Hidden Trick
Presenter: SyberMath

1 Problem

Given the relation

$$\left(\sqrt{2+\sqrt{3}}\right)^x + \left(\sqrt{2-\sqrt{3}}\right)^x = 4, \quad (1)$$

solve for all real values of x .

Note: See my write-up on the **unipodal algebra** for reference.

2 Solution

First, I'll rewrite the Given in the form

$$(2+\sqrt{3})^{x/2} + (2-\sqrt{3})^{x/2} = 4, \quad (2)$$

and then set

$$y = x/2, \quad (3)$$

to get

$$(2+\sqrt{3})^y + (2-\sqrt{3})^y = 4. \quad (4)$$

Next, I set

$$a = (2 + \sqrt{3})^y u_+ + (2 - \sqrt{3})^y u_-. \quad (5)$$

This gives us:

$$aa^- = [(2 + \sqrt{3})(2 - \sqrt{3})]^y = 1^y = 1. \quad (6)$$

This should be a very useful result for the near future.

Now we return to (5) and rewrite it into standard-basis form:

$$a = \frac{1}{2}[(2 + \sqrt{3})^y + (2 - \sqrt{3})^y] + \frac{1}{2}[(2 + \sqrt{3})^y - (2 - \sqrt{3})^y]u. \quad (7)$$

On using the Given information in (1) and setting

$$B \equiv (2 + \sqrt{3})^y - (2 - \sqrt{3})^y, \quad (8)$$

then (7) becomes

$$a = 2 + \frac{1}{2}Bu. \quad (9)$$

From this we get that

$$aa^- = 4 - \frac{1}{4}B^2. \quad (10)$$

So, combining this last result with (6), we get

$$4 - \frac{1}{4}B^2 = 1. \quad (11)$$

Solving this for B , we have that

$$B = \pm 2\sqrt{3}. \quad (12)$$

On combining this with (8), we get

$$(2 + \sqrt{3})^y - (2 - \sqrt{3})^y = \pm 2\sqrt{3}. \quad (13)$$

By adding this result with the Given (1) and simplifying, we get

$$(2 + \sqrt{3})^y = 2 \pm \sqrt{3}. \quad (14)$$

Setting $y = 1$ gives us the plus sign on the RHS, and setting $y = -1$ gives us the minus sign on the RHS.

Thus, using (3), we get two solution for x :

$$x = 2, -2. \quad (15)$$