Math Diversion 1067

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> Truth, like oil, will in time rise to surface.
> — Charlie Chan

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Source:  https://www.youtube.com/watch?v=it6kW89PpI0
Title: This Broke My Brain
 Presenter: SyberMath
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1 Problem

Given the relation

$$7^{6-x} = x + 2\,, \tag{1}$$

solve for all values of x.

Hint: This looks like a job for the Lambert W function.

For a reference on the Lambert W function (especially how I use it), see my PDF write-up on it.

2 Solution

In cases like this, I have found it best to introduce a variable substitution. So, let

$$y = x + 2 \quad \text{then} \quad x = y - 2\,. \tag{2}$$

Thus (1) becomes, with some algebra,

$$7^8 = y7^y\,. \tag{3}$$

Now, on taking the Lambert W function base 7, we get

$$y = W_{(7)}(7^8) = \frac{W_n(7^8 \ln 7)}{\ln 7}\,, \tag{4}$$

where $n \in \mathbb{Z}$. Thus, we have that

$$x = \frac{W_n(7^8 \ln 7)}{\ln 7} - 2\,. \tag{5}$$