Math Diversion 1068

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April 10, 2026

`Source: https://www.youtube.com/watch?v=AFU4Q7TIXU4`
`Title: Tricky Maths Question for Competitive Exams`
`Presenter: Math Beast`

1 Problem

Given the relation

$$\frac{x}{y} + \frac{y}{x} = x + y \quad \text{where} \quad x, y \in \mathbb{Z}^+ \,, \tag{1}$$

solve for all allowed values of x, y.

2 Solution

It seems reasonable enough to try a solution by inspection, and get $x = y = 1$, but I prefer a systematic approach, especially since x, y are positive-integer valued. Let

$$\frac{x}{y} \equiv \eta \,, \tag{2}$$

then (1) becomes

$$\eta + \eta^{-1} = x + y = y(\eta + 1) \,. \tag{3}$$

On solving this for y, we have that

$$y = (\eta + \eta^{-1})/(\eta + 1) \in \mathbb{Z}^+ \,, \tag{4}$$

because $y \in \mathbb{Z}^+$. This time, by inspection, we get

$$\eta = 1 \quad \text{so that} \quad y = 1, \quad \text{hence} \quad x = y\eta = 1 \,. \tag{5}$$

What about uniqueness? Clearly, if we consider η to be a rational, non-integer number, then, because x and y enter (1) symmetrically, we must also get a solution on making the replacements $x \leftrightarrow y$ and $\eta \leftrightarrow \eta^{-1}$:

$$x = (\eta^{-1} + \eta)/(\eta^{-1} + 1) \in \mathbb{Z}^+ \,, \tag{6}$$

which again will only allow the solution $\eta = 1$ with $x = y = 1$. Hence our previous solution is unique.