Math Diversion 1069

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A clue is anything that doesn't happen
the way it oughtta happen.
— Harry Orwell, TV
show *Harry O*

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Source: https://www.youtube.com/watch?v=WgvrsAF3pEw
Title: Cambridge Maths Interview Question | 95% got it wrong
 Presenter: Math Beast
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1 Problem

Given the relation

$$(-2)^c = 2\,, \tag{1}$$

solve for all allowed complex values of c.

2 Solution

There's no unique way to solve this for c, so I'll just do it my way. First, let's rewrite the Given as

$$(-2)^c = -(-2)\,, \tag{2}$$

hence,

$$(-2)^{c-1} = -1\,. \tag{3}$$

But

$$-1 = e^{\pi i} \quad \text{and} \quad 1 = e^{2\pi ik}\,, \tag{4}$$

where $k \in \mathbb{Z}$, so we get

$$(-2)^{c-1} = e^{\pi i + 2\pi ik} = e^{\pi i(1+2k)}\,. \tag{5}$$

On taking the natural logarithm, we have that

$$(c-1)\ln(-2) = \pi i(1+2k)\,, \tag{6}$$

or

$$c = \frac{\pi i(1+2k)}{\ln(-2)} + 1\,. \tag{7}$$

But we can write

$$\ln(-2) = \ln(-1) + \ln(2) = \ln(e^{\pi i}) + \ln(2) = \pi i + \ln(2)\,. \tag{8}$$

Substituting this result into (7), we get

$$c = \frac{\pi i(1+2k)}{\pi i + \ln(2)} + 1\,, \tag{9}$$

and finally,

$$c = \frac{\pi(1+2k)}{\pi - i\ln(2)} + 1 = \frac{\pi(1+2k) + (\pi - i\ln(2))}{\pi - i\ln(2)} = \frac{2\pi(1+k) - i\ln(2)}{\pi - i\ln(2)}\,, \tag{10}$$

which is what WolframAlpha got.