Math Diversion 1081

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April 28, 2026

Success is failing 19 times and succeeding the 20th.
— Julia Andrews

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Source:  https://www.youtube.com/watch?v=1HVUZP69W0c
Title: Solving Logarithmic Equations with Different Bases Made Easy!
 Presenter: L.JARED2
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1 Problem

Given the relation

$$\log_4(6-x) = \log_2 x\,, \tag{1}$$

find the real solutions of x.

2 Solution

We start by raising 4 to the power of Eq. (1), getting

$$4^{\log_4(6-x)} = 4^{\log_2 x} = 2^{2\log_2 x} = 2^{\log_2 x^2}\,, \tag{2}$$

which, after simplifying becomes

$$6 - x = x^2\,. \tag{3}$$

Placing this in standard form, we have that

$$x^2 + x - 6 = 0\,. \tag{4}$$

On factoring this we get

$$(x+3)(x-2) = 0\,, \tag{5}$$

which has roots

$$x = -3 \quad \text{and} \quad x = 2\,. \tag{6}$$

Now, $x = -3$ won't work because $\log_2(-3)$ is not real, but $\log_4(6-(-3))$ is real. So the answer is $x = 2$.