Math Diversion 1083

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> The human mind has never invented a labor-saving
> machine equal to algebra.
> — J. Willard Gibbs

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Source:  https://www.youtube.com/watch?v=P66nkc3bKrE
Title: find the value of x ll maths olympiad
 Presenter: maths curiosity
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1 Problem

Given the relation

$$\sqrt{x+5}+\sqrt{x-5}=5\,, \tag{1}$$

find the real solutions for x.

2 Solution

I shall be using the unipodal numbers to solve this problem, but it can be solved conventionally as well.

Let

$$a=\sqrt{x+5}u_{+}+\sqrt{x-5}u_{-}=5\,, \tag{2}$$

then

$$a^2=(x+5)u_{+}+(x-5)u_{-}=5\,. \tag{3}$$

Upon converting a^2 to standard basis, we get

$$a^2=x+5u\,. \tag{4}$$

Returning to (2), and converting it to standard basis, we have that

$$a=\tfrac{1}{2}[\sqrt{x+5}u_{+}+\sqrt{x-5}]+u\tfrac{1}{2}[\sqrt{x+5}-\sqrt{x-5}]u=5\,, \tag{5}$$

from which, with

$$B\equiv\sqrt{x+5}-\sqrt{x-5}\,, \tag{6}$$

we have that

$$a = \frac{5}{2} + \frac{B}{2}u\,. \tag{7}$$

On squaring this equation, we have that

$$a^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{B}{2}\right)^2 + \frac{5B}{2}u\,. \tag{8}$$

So, what to do now? Well, if we can solve for B, we can combine (6) and (1) to solve for x. So let's do that! Now, on equating the unipotent parts of (4) and (7), we get

$$B = 2\,. \tag{9}$$

So, on adding (6) and (1), and using (9), we have that

$$x = \frac{29}{4} = 7.25\,. \tag{10}$$