Math Diversion 1085

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A man convinced against his will is unconvinced still.
— Proverb

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Source:   https://www.youtube.com/watch?v=z2Yf5nQY5Kc
Title: Can You Solve This Harvard University
 Interview Question
 Presenter: HkLogics
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1 Problem

Given the relation

$$(x/2)^6 = 6^6\,, \tag{1}$$

solve for x in the complex numbers.

2 Solution

From the start, we can expect six complex solutions because x is effectively to the sixth power, and so has six complex roots.

Let's transform (1) by a series of algebraic steps:

$$\left(\frac{1}{6}\right)^6 \left(\frac{x}{2}\right)^6 = 1\,, \tag{2}$$

and then to

$$\left(\frac{x}{12}\right)^6 = 1\,. \tag{3}$$

Now, to access the full set of complex roots, we need to make the standard substitution:

$$1 = e^{2\pi ik} \quad \text{where } k \in [0..5]\,. \tag{4}$$

So, on substituting this into (3), we have that

$$\left(\frac{x}{12}\right)^6 = e^{2\pi ik} \quad \text{where } k \in [0..5]\,. \tag{5}$$

The next step is obvious: We take the sixth root across the equation, to get

$$\frac{x}{12} = e^{2\pi i k/6} \quad \text{where } k \in [0..5]\,. \tag{6}$$

Lastly, we just list the roots individually:

$$\{x\} = \{12,\ 12e^{\pi i/3},\ 12e^{2\pi i/3},\ 12e^{\pi i},\ 12e^{4\pi i/3},\ 12e^{5\pi i/3}\}\,. \tag{7}$$

As a final touch up, you could replace $12e^{\pi i}$ by -12.