Math Diversion 1087

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CONCERNING HORSES

I've been through the desert
On a horse with no name
It felt good to be out of the rain
In the desert, you can remember your name
'Cause there ain't no one for to give you no pain.
— 'A Horse with No Name' (*America*, 1971)

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Source: https://www.youtube.com/watch?v=m9N5S3Hq30o
Title: Simplify If You Can!
 Presenter: aplusbi
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1 Problem

Given the relation

$$\phi = \frac{e^{i\theta/2}}{e^{i\theta}+1}\,, \tag{1}$$

simplify ϕ.

2 Solution

Multiply numerator and denominator by $e^{-i\theta/2}/2$:

$$\phi = \frac{1/2}{(e^{i\theta/2}+e^{-i\theta/2})/2}\,, \tag{2}$$

from which we get

$$\phi = \frac{1}{2}\frac{1}{\cos(\theta/2)}\,. \tag{3}$$

You can prove that

$$\cos \tfrac{1}{2}\theta = \frac{e^{i\theta/2} + e^{-i\theta/2}}{2}\,, \tag{4}$$

by employing Euler's Formula

$$e^{i\theta/2} = \cos \tfrac{1}{2}\theta + i \sin \tfrac{1}{2}\theta\,, \tag{5}$$

and a bit of complex algebra. Hint: replace i by $-i$ to get an additional equation.